EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SPECIAL REPEAT EXAMINATION IN SCIENCE -2007/2008

THIRD YEAR, FIRST AND SECOND SEMESTER (Feb, 2010)

MT 307 - CLASSICAL MECHANICS III

Answer all Questions | Time: Three hours

1. A projectile located at colatitude λ is fixed with velocity in a southward direction at an angle α with the horizontal.

   (a) Find the position of the projectile after time t.

   (b) Prove that after time t the projectile is deflected toward the east of the original vertical plane of motion by the amount

   $$\frac{1}{3}\omega g \sin\lambda t^3 - \omega v_0 \cos(\alpha - \lambda)t^2.$$

   (c) Prove that when the projectile returns to the horizontal, it will be at the distance

   $$\frac{4\omega v_0^3 \sin^2\alpha}{3g^2}\{3\cos\alpha\cos\lambda + \sin\alpha\sin\lambda\}.$$

2. (a) Define the following terms:

   i. linear momentum;

   ii. angular momentum;

   iii. moment of force.

   (b) Prove the followings:

   i. the rate of change of total angular momentum $\underline{H}$ about a point O is equal to the total moment of the external forces about O,

ii. $\underline{H} = \underline{r}_G \wedge M\underline{v}_G + \underline{H}_G$,

where $\underline{H}_G$ is the angular momentum about the centre of mass and $\underline{v}_G$ is the velocity of the centre of mass.

(c) A uniform rod is placed on a horizontal table with two thirds of its length having over the edge of the table. If the rod is at right angle to the edge and is released then show that it will begin to slip when the rod has turned through an angle $\tan^{-1}(\mu/2)$ where μ is the coefficient of friction between the rod and the table.

3. (a) Prove that with the usual notation the moment of inertia of a body about any line through the origin of a coordinate frame is,

$$I = Al^2 + Bm^2 + Cn^2 - 2mnF - 2nlG - 2lmH.$$

(b) A pendulum consists of a rod of length $2a$ mass m and a spherical bob of radius $a/3$ mass $15m$.One end of the rod is attached to the roof and the other end is attached to the spherical bob. Find the moment of inertia of the pendulum:

i. about an axis through the other end of the rod and at right angles to the rod,

ii. about a parallel axis through the centre of mass of the pendulum, given that the centre of mass of the pendulum is $a/12$ from the centre of the sphere.

4. (a) With the usual notations, state the Euler's equation for the motion of a rigid body with one point fixed.

(b) Show that the kinetic energy and angular momentum of the torque free motion of a rigid body is constant.

(c) A body moves freely about a point O. The principal moment of inertia at O being $6A, 3A, A$. Initially the angular velocity of the body has components $\omega_1 = n, \omega_2 = 0, \omega_3 = 3n$ about the principal axis. Show that after time t, $\omega_2 = -\sqrt{5}n \tanh \sqrt{5}nt$.

5. (a) With the usual notations, state the Lagrange's equation for the impulsive motion from the Lagrange's equation for a holonomic system.

   (b) A square $ABCD$ formed by four equal rods, each of length $2l$ and mass m joined smoothly at their ends, rests on a smooth horizontal table. An impulse of magnitude I is applied to the vertex A in the direction of AD.

      i. Find the equation of motion of the frame.

      ii. Show that the kinetic energy of the square immediately after the application of impulse is $\dfrac{5I^2}{16m}$.

6. (a) Define the Poisson Bracket.

   With the usual notations show that

$$\frac{dF}{dt} = [F, H] + \frac{\partial F}{\partial t}$$

   for any function $F = F(p_j, q_j, t), \quad j = 1, 2, \cdots, n$.

   Prove that if F and G are constants of motion then $[F, G]$ is also a constant of motion.

   (b) For a certain system with two degree of freedom, the hamiltonian is given by

$$H = \eta^2(p_1^2 + p_2^2) + \nu^2(p_1 q_1 + p_2 q_2)^2$$

   where η and ν are constants.

   Show that if H is a constant and $F = p_1 q_1 + p_2 q_2$ then

$$[F, H] = 2(H - \nu^2 F^2).$$