



EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE -2008/2009

FIRST YEAR, SECOND SEMESTER (Jan./Mar., 2011)

EXTMT 105 - THEORY OF SERIES

(PROPER & REPEAT)

Answer all Questions

Time: One hour

1. (a) Define what is meant by the convergent or divergent of an infinite series $\sum_{n=1}^{\infty} a_n$.

Show that the series

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \dots$$

is convergent and find its sum.

- (b) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of real numbers.

- i. Show that $a_n \rightarrow 0$ as $n \rightarrow \infty$ if $\sum_{n=1}^{\infty} a_n$ is convergent.

Is the converse of the above statement true? Justify your answer.

- ii. Is it true that, if $\sum_{n=1}^{\infty} (a_n + b_n)$ is convergent implies $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent? Justify your answer.

(a) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of non-negative real numbers.

i. If $a_n \leq kb_n$ for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} b_n$ is convergent, show that $\sum_{n=1}^{\infty} a_n$ is convergent, where k is a positive real number.

ii. If $\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}$, for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} b_n$ is convergent, then prove that

$\sum_{n=1}^{\infty} a_n$ is convergent.

(b) Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers. Prove that, if $\sum_{n=1}^{\infty} |a_n|$ is convergent then

$\sum_{n=1}^{\infty} a_n$ is also convergent.

Is the converse of the above statement true? Justify your answer.