EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009

FIRST YEAR, FIRST SEMESTER (July/August, 2010)

EXTMT 101 - FOUNDATION OF MATHEMATICS

Answer all questions Time : Three hours

1. (a) Prove the following equivalences using the laws of algebra of propositions:

   i. $(p \wedge q) \vee \neg p \equiv \neg p \vee q$;

   ii. $[p \wedge (q \vee r)] \wedge \neg[(\neg q \vee \neg r) \wedge r] \equiv p \wedge q$.

   (b) Test the validity of the argument "If you are a mathematician then you are clever. You are clever and rich. Therefore if you are rich then you are a mathematician".

   (c) In a group of students in a school at least 70% study Applied Mathematics, at least 75% study Pure Mathematics, at least 80% study Chemistry and at least 85% study Physics. Find at least what percentage of students study all four subjects.

2. (a) For any sets A, B and C, prove that:

   i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$;

   ii. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

   (b) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then show that $B = C$. Hence prove that

   $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$ if and only if $A \triangle (B \cup C) = (A \triangle B) \cup (A \triangle C)$.

3. (a) Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \ / \ x \neq 0, \ y \neq 0\}$ and define a relation R on S by $(x_1, y_1)R(x_2, y_2) \Leftrightarrow x_1y_1(x_2^2 - y_2^2) = x_2y_2(x_1^2 - y_1^2)$

   i. Show that R is an equivalence relation.

   ii. If (a, b) is a fixed element of S, show that $(x, y)R(a, b) \Leftrightarrow \dfrac{y}{x} = \dfrac{b}{a}$ or $\dfrac{y}{x} = \dfrac{-a}{b}$

   (b) If R_1 and R_2 are equivalence relations on a set X, prove the following:

   i. $R_1 \cap R_2$ is an equivalence relation;

   ii. $R_1 \cup R_2$ need not be an equivalence relation.

4. (a) Define each of the following terms:

   i. injective mapping;

   ii. surjective mapping;

   iii. inverse mapping.

   (b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x + 1, & \text{if } x \geq 0; \\ x, & \text{if } x < 0. \end{cases} \quad \text{and } g(x) = \begin{cases} 3x, & \text{if } x \geq 0; \\ x + 3, & \text{if } x < 0. \end{cases}$$

   Find the formula for $g \circ f$.

   Show that $g \circ f$ is a bijection and give a formula for $(g \circ f)^{-1}$

5. (a) Define the following terms:

   i. partially ordered set;

   ii. totally ordered set;

   iii. first element;

   iv. minimal element.

   (b) Show that every partially ordered set has at most one first element.

   (c) Show that first element of every partial ordered set is a minimal element.

   Is the converse true? Justify your answer.

   (d) Prove that in a totally ordered set every minimal element is a first element.

6. (a) Show that the square of any integer is of the form $3k$ or $3k + 1$, where k is an integ

   (b) Using the Euclidean algorithm find the gcd(1819,3587) and hence express the gcd(1819,3587) as a linear combination of 1819 and 3587.

   (c) A customer bought 12 pieces of fruit, apples and oranges, for Rs. 132. If an ap costs Rs. 3 more than an orange and more apples than oranges were purchased, h many pieces of each kind were bought.