EASTERN UNIVERSITY, SRI LANKA

Faculty of Commerce and Management

Department of Commerce

First Year / First Semester Examination in Business Administration / Commerce 2002/2003 & 2002/2003(A).

COM 1023 Mathematics and Statistics

Answer All questions . Time: Three (03) Hours

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1. (a) Solve the following equations.

   i. $x^2 - \frac{11}{2}x - 6 = 0$

   ii. $2^{4x+3} - 9 \times 2^{2x+1} + 4 = 0$

   iii. $6^{3-2x} \times 5^{x+4} = 72$

   (b) The cost function $C(x)$, is the cost of producing x units of certain commodity, and is assumed to be linear, say

   $$C(x) = mx + b\,,$$

   where m is the marginal cost and b is the fixed cost. Suppose the marginal cost is Rs. 6.25 and it costs Rs. 500 to produce 25 units.

i. Find the equation for the cost function $C(x)$.

ii. Compute the fixed cost.

(c) A calculator manufacturing company determines that the demand function of their calculators is

$$x = 400(50 - t) ,$$

where t is the wholesale price per calculator and x is the number of calculators the company can sell at that price. Assume that the company's fixed cost is Rs. 8000 and material and labour costs per calculator is Rs. 8.

i. Determine the total cost function $C(x)$ in terms of t .

ii. Determine the total revenue function $R(x)$ in terms of t .

iii. Determine the profit function $P(x)$ in terms of t .

iv. Determine the price that yields the maximum profit.

v. Estimate the maximum profit

2. (a) i. The amount p is invested at the rate of interest r . If the interest compounded annually, show that the amount after t years, $A(t)$, is

$$A(t) = p(1+r)^t$$

ii. On February 15, 1986 Rs. 10,000 and on February 15, 1987 Rs. 25,000 were invested in a bond. The value of the bond on February 15, 1988 was Rs. 40,125. Assume that the rate

of interest was constant over that two years period and the interest was paid annually. What was the rate of interest?

(b) A woman invests Rs. 10,000 in an account that pays 8% interest compounded annually. At the end of each year she withdraws Rs. 300 .

i. Find the recurrence relation for the amount of money in her account after n years.

ii. Solve the equation obtained in (i) using the initial investment $P_o =$ Rs. 10,000 .

iii. How much money will be in her account after 10 years?

(c) A ball is dropped to the ground from a height of $60m$ and each time when it bounces it goes $2/3$ as high as it was previously. How high does it go on the 10 th bounce?

(a) i. Prove that

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1},$$

where n is a real number.

ii. Find the limits of the following :

A. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

B. $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$

C. $\lim_{x \to \infty} \frac{x^2 - 1}{x^3 - x}$

D. $\lim_{x \to 3} \frac{x^{-5} - 3^{-5}}{x^{-2} - 3^{-2}}$

(b) i. Differentiate the following with respect to x

A. $y = 5x^4 - 3x^{-2} + 5$

B. $y = \dfrac{x+1}{x-1}$

ii. A company's cost function for a certain item, through its experience, is estimated by

$$C(x) = 3x^2 - 40x + 2500$$

and the revenue function is

$$R(x) = 200x\,.$$

Using differentiation find the number of units that the company should produce to maximize the profit.

(c) Integrate the following:

i. $\int (x+5)^2 dx$

ii. $\int \dfrac{x^2-1}{x+1} dx$

iii. $\int \dfrac{5x+6}{(x-1)(x+2)} dx$

iv. $\int \dfrac{4x+1}{2x^2+x+1} dx$

4. (a) i. Define the following terms.

A. Event

B. Trial, in an experiment.

ii. A box of 100 items contains 5 defective items. To test the box a buyer selects a random sample of 6 items. What is the

probability that the sample contains at least one defective item?

(b) If an investor randomly select two utility stocks with replacement from a group of three, the probability of selecting the two with the greatest growth potentials is $1/3$.

i. What is the probability that the first stock chosen will be the one of the two stocks that have greatest growth potentials? Label as event A, the event associated with this probability.

ii. Given that event A has occurred, what is the probability that the second stock chosen will be the one with greatest growth potentials of the two remaining stocks?

iii. What is the probability of selecting the two stocks with the greatest growth potentials?

(c) A department store is preparing to adopt a new credit management policy in an attempt to reduce the number of credit customers defaulting on their payments. The credit manager has suggested that, in the future, credit should be discontinued to any customer who has twice been a week or more late with his monthly installment payment. The manager supports her claim by noting that past credit records show that 90% of all those defaulting on their payments were late with at least two monthly payments. Suppose from our own investigation we have found that 2% of all credit customers actually default on their payments and that 45% of those who have not defaulted have had at least two late monthly

payments. Find the probability that a customer with two or more late payments will actually default on his payments.

(a) The following set of data represents the weights of 30 people in a company.

100.6	84.2	85.2	91.0	85.5	94.7	101.7	87.2	92.7	98.3
73.6	87.6	110.4	105.8	103.7	93.7	89.0	73.1	90.0	68.1
91.6	95.9	94.7	79.1	93.7	79.4	84.5	94.2	88.6	97.8

i. Construct a frequency histogram for the above data using the class width of 5.

ii. What proportion of the observations are less than 78.5?

iii. If one observation is chosen at random from this set of data, what is the probability that the observation will be greater than or equal to 102.5?

(b) The marks of 50 students in a subject are given on the frequency distribution below.

Class Interval	Frequency
01 - 10	1
11 - 20	4
21 - 30	6
31 - 40	9
41 - 50	12
51 - 60	8
61 - 70	5
71 - 80	3
81 - 90	1
91 - 100	1

i. Find the mean, median and mode of the distribution.

ii. Find the cut - off marks for the award of scholarship to 5% of the students.

iii. Estimate P_{40} and interpret your answer.

iv. Estimate PR_{40} and interpret your result.

v. Compute the standard deviation of the distribution.