EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE SECOND EXAMINATION IN SCIENCE (2002/2003)

SECOND SEMESTER (Oct./Nov., 2007)

EXTMT 217 - MATHEMATICAL MODELING

Answer all questions

Time : Two hours

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1. Write down the steps involved in a mathematical model building process.

   Sate the Newton's law of cooling.

   At 1.00 pm, Mary puts into a refrigerator a can of soda that has been sitting in a room of temperature $70°F$. The temperature in the refrigerator is $40°F$. Fifteen minutes later, at 1 : 15 pm; the temperature of the soda has fallen to $60°F$. At some later time, Mary removes the soda from the refrigerator to the room, where at 2 : 00 pm the temperature of the soda is $60°F$. At what time did Mary remove the soda from the refrigerator?

2. (a) Explain the logistic model

$$\frac{dp}{dt} = ap - bp^2, \quad p(t_0) = p_0$$

   of the population growth of a single species.

   Prove that $\dfrac{a - bp_0}{a - bp(t)}$ is positive for $t_0 < t < \infty$ where $p(t_0) = p_0$.

   Find $p(t)$ and the limiting value of $p(t)$, $t > t_0$.

(b) A man eats a diet of 2500 cal/day, 1200 of them go to basal metabolism (i.e., get used up automatically). He spends approximately 16 cal/kg/day times his body weight (in kilograms) in weight proportional exercise. Assume that the storage of calories as fat is 100% efficient and that 1 kg fat contains 10000 cal. Find how his weight various with time.

3. Suppose a x–force and a y-force are engaged in combat. Let $x(t)$ and $y(t)$ denote the respective strength of the forces at time t, when t is measured in days from the start of the combat. Conventional combat model is given by

$$\frac{dx}{dt} = -ax(t) - by(t) + P(t)$$
$$\frac{dy}{dt} = -dy(t) - cx(t) + Q(t).$$

Explain the terms involved in these equations.

By assuming that there is no reinforcement arrived and no operational losses occur, obtain a simplified model and sketch the graph. Hence show that

$$x(t) = x_0 \cosh(\beta t) - \gamma y_0 \sinh(\beta t),$$

where $\beta = \sqrt{bc}$ and $\gamma = \sqrt{b/c}$.

4. Consider n vehicles traveling in a straight line. If $V_n(t)$ is the speed of n^{th} vehicle at time t, obtain the model

$$\frac{d}{dt}V_{n+1} = V_n(t) - V_{n+1}(t).$$

Interpret this equation and show that

$$V_{n+1}(t) = \frac{1}{(n+1)!}\int_0^t u^{n-1}e^{-u}V_1(t-u)du$$

where $V_1(t)$ is the speed of the lead vehicle.

Suppose the lead vehicle is standing still at $t = 0$ and acquires a constant cruising speed V_c for $t > 0$.

Show that

$$V_{n+1}(t) = V_c G_n(t),$$

where $G_n(t) = 1 - e^{-t}\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + ... + \frac{t^{n-1}}{(n-1)!}\right)$.

Show that there is no possibility of collision in this model.