



EASTERN UNIVERSITY, SRI LANKA
EXTERNAL DEGREE EXAMINATION IN SCIENCE
FIRST YEAR FIRST SEMESTER -2003/2004
(Oct./ Nov., 2006)
MT 106 - TENSOR CALCULUS
Proper & Repeat

Answer all questions

Time : One hour

1. (a) Write the transformation equation for the following tensors:

i. A_{qr}^{ms} ,

ii. B_{mn}^{pqr} ,

iii. C_{ijk} .

(b) Let A_{rst}^{pqr} be a tensor.

i. Choose $p = t$ and show that A_{rst}^{pqr} , where the summation convention is employed, is a tensor. What is its rank?

ii. Choose $p = t$ and $q = s$ and show similarly that A_{rst}^{pqr} is a tensor. What is its rank?

(c) Find g and g^{jk} corresponding to the line element

$$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3.$$

2. (a) Define the following:

- i. Christoffel symbols of the first and second kind;
- ii. Geodesic.

(b) Determine the Christoffel symbols of the second kind for the line element

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

and find the corresponding geodesic equations.

(c) With the usual notations, prove the following:

- i. $\frac{\partial g_{rs}}{\partial r^m} = [rm, s] + [sm, r],$
- ii. $\frac{\partial g^{rs}}{\partial r^m} = -g^{rn} \Gamma_{mn}^s - g^{sn} \Gamma_{mn}^r.$

Hence deduce that the covariant derivative of metric tensors g^{r^k} and g_{r^k} are zero.