EASTERN UNIVERSITY, SRI LANKA

FIRST YEAR EXAMINATION IN SCIENCE, 2002/2003

EXTERNAL DEGREE

SECOND SEMESTER

(Sept./Oct. '2005)

EXTMT 105 - THEORY OF SERIES

Answer All Questions Time: 1 Hour

Q1. (a) Define what is meant by the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent.

[5 Marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+4)} = \frac{1}{1.5} + \frac{1}{2.6} + \frac{1}{3.7} + \dots$$

is convergent and find its sum. [30 Marks]

(b) State the theorem of **Integral Test**.

[10 Marks]

By using the above theorem or otherwise, for the following cases of $p \in \mathbb{R}$,

(i) $p > 1$,

(ii) $p = 1$,

(iii) $0 < p < 1$,

determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges or diverges.

[15 Marks]

(c) State the theorem of **Alternating Series Test**.

[10 Marks]

Use the above theorem to decide whether the following series converge or diverge:

(i) $\sum_{n=1}^{\infty} \sin\left(\frac{(n^2+1)\pi}{n}\right)$;

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n+1}{n(n+2)}\right)$.

[30 Marks]

Q2. (a) For the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$, find the interval and radius of convergence.

[25 Marks]

(b) (i) Let $f_n, f : A \subseteq \mathbb{R} \to \mathbb{R}$. Define what is meant by $f_n \to f$ as $n \to \infty$ uniformly on A.

[5 Marks]

(ii) Let $f_n, f : A \subseteq \mathbb{R} \to \mathbb{R}$. If $f_n \to f$ uniformly on A as $n \to \infty$ and each f_n, $n \in \mathbb{N}$ is continuous on A, then prove that f is continuous on A.

[20 Marks]

(iii) Let $f_n, f : [a,b] \subseteq \mathbb{R} \to \mathbb{R}$ and let $f_n \to f$ uniformly on $[a,b]$ as $n \to \infty$ and each f_n, $n \in \mathbb{N}$ is continuous on $[a,b]$. Show that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \int_a^b f_n(x)\,dx.$$

[20 Marks]

(c) (i) Show that

$$\ln x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n} \quad \text{for } |x-1| < 1.$$

[15 Marks]

(ii) Use the result in part(i) and the **Abel**'s theorem to show that

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.$$

[15 Marks]