EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 1996/97

(June/July' 2004)

EXTERNAL DEGREE

EXMT 203 & 204 - EIGENSPACES AND QUADRATIC FORMS & DIFFERENTIAL GEOMETRY

Answer four questions only selecting two questions from each section

Time : Two hours

Section A

1. Define the term "an eigenvalue of a linear transformation".

   (a) Prove that an $n \times n$ square matrix A is similar to a diagonal matrix D whose diagonal elements are the eigenvalues of A if, and only if A has n linear independent eigenvectors.

   (b) Prove that the eigenvalues of a Hermitian matrix are real.

   (c) Let

$$A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{pmatrix}.$$

   Find a non-singular matrix P such that $P^{-1}AP$ is diagonal.

2. Define the minimum polynomial of a square matrix.

(a) State and prove the Cayley-Hamilton theorem.

(b) Let $m(t)$ be a minimum polynomial of a matrix A and $f(t)$ be a polynomial such that $f(A) = 0$. Show that $m(t)$ divides $f(t)$.

(c) Find the minimum polynomial of A given by

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}.$$

3. (a) Find an orthogonal transformation which reduces the following quadratic form to a diagonal form

$$5x_1^2 + 6x_2^2 + 7x_3^2 - 4x_1x_2 + 4x_2x_3.$$

(b) Show that one of the two quadratic forms given below is positive definite and find a non-singular linear transformation which reduces this to a unit form and the other to a diagonal form.

$\phi_1 = x_1^2 + 2x_2^2 + 8x_2x_3 + 12x_1x_2 + 12x_1x_3$

$\phi_2 = 3x_1^2 + 2x_2^2 + 5x_3^2 + 2x_2x_3 - 2x_1x_3.$

Section B

4. State and prove Serret-Frenet formulae.

Let C be a curve of constant torsion τ. P is any point on the curve C. Point Q is taken at a constant distance c from P on the binormal to C at P. Show that the angle between the binormal to the locus of Q and the binormal of the given curve is $\tan^{-1}\left(\dfrac{c\tau^2}{\kappa\sqrt{1+c^2\tau^2}}\right)$.

5. What is meant by saying that a curve is a helix?

Prove, with the usual notation, that a necessary and sufficient condition for a curve to be a helix is that $\dfrac{\tau}{\kappa}=$ constant.

Show that the curve $r(\theta) = (\ a\cos\theta,\ a\sin\theta,\ a\theta\cot\beta\ )$ is a helix, where a and β are constants.

6. Define the term " Osculating sphere of a space curve" and find its radius and center.

Show that the tangent, principal normal and binormal to the locus C_1 of the center of the osculating sphere of a given curve C are parallel to the binormal, principal normal and tangent to C respectively at the corresponding points.