EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE 1996/97

(June/July' 2004)

FIRST SEMESTER

EXTERNAL DEGREE

EXMT 101 - FOUNDATION OF MATHEMATICS

Answer all questions

Time : Three hours

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1. (a) i. Define the following terms "tautology" and "contradiction" as applied to a proposition.

   ii. Explain what is meant by the statement that two propositions are logically equivalent.

   (b) Let p, q and r be three propositions. Determine whether each of the following statement is a tautology.

   i. $[(p \wedge q) \wedge (q \vee r) \wedge > r] \longrightarrow p$;

   ii. $[(p \leftrightarrow q) \wedge (> p \vee r) \wedge > r] \longrightarrow > p$.

   (c) Test the validity of the following argument:

   On my girlfriend's birthday, I bring her flowers.
   Either it's my girlfriend's birthday or I come late from office.
   I did not bring flowers today. Therefore, today I come late from office.

2. Let A, B be any subsets of a Universal set X. Define the sets

- $A \setminus B$,
- $A \triangle B$.

Let A, B, C be three subsets of a Universal set X. Prove the following:

(a) $A \cup B = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$;

(b) $(A \cap B) \setminus (A \cap C) = A \cap (B \setminus C)$;

(c) $(A \triangle B) = (A \cup B) \setminus (A \cap B)$;

(d) $(A \triangle B) \cap (A \cap B) = \phi$.

3. What is meant by an equivalence relation?

(a) Let $\mathbb{R}$ be the set of all real numbers. A relation ρ is defined on $\mathbb{R}^2$ by

$$(a, b)\rho(c, d) \Leftrightarrow a + d = b + c.$$

Show that ρ is an equivalence relation.

Is it true that a relation defined on $\mathbb{Z}$ as $x \; \rho \; y \Leftrightarrow$ "x divedes y" an equivalence relation.

(b) Let A be a set and let $\sim$ be an equivalence relation on A. Let $[a] = \{x \in A \mid x \sim a\}$. Prove the following:

i. $[a] \neq \Phi \quad \forall \, a \in A$;

ii. $a \sim b \Leftrightarrow [a] = [b] \quad \forall \, a, b \in A$;

iii. $b \in [a] \Leftrightarrow [a] = [b] \quad \forall \, a, b \in A$;

iv. Either $[a] = [b]$ or $[a] \cap [b] = \Phi$.

4. (a) Define the following terms as applied to a mapping.

   i. Injective,

   ii. Surjective,

   iii. Bijective.

   (b) Let $f : A \longrightarrow B$ and $g : B \longrightarrow A$ be two mappings such that $g \circ f = I_A$ and $f \circ g = I_B$. Prove that f is bijective and $g = f^{-1}$.

   (c) Let S and T be two sets and let $f : S \longrightarrow T$ be a mapping. Prove that

   i. if f is injective then $f(A \cap B) = f(A) \cap f(B)$ for all $A, B \subseteq S$;

   ii. f is injective if and only if $f(A) \cap f(S \setminus A) = \Phi \ \forall A \subseteq S$.

5. (a) Define the following terms:

   - Partially ordered set,
   - Totally ordered set.

   Let $A = \{2^n \mid n \in \mathbb{N}\}$. Define a relation $\preceq$ on A as $a \preceq b$ if and only if a divides b for $a, b \in A$. Prove that $(A, \preceq)$ is a totally ordered set.

   (b) Define the following elements of a partially ordered set.

   - First element,
   - Last element,
   - Minimal element.

   i. Show that every partially ordered set has at most one first element and at most one last element.

   ii. Show that if a totally ordered set has minimal element, then it will be the first element.

6. Define the following:

- The greatest common divisor (gcd) of two integers a and b ,
- Prime number,
- The least common multiple (lcm) of two integers a and b.

(a) Prove that if $p|ab$, where p is a prime number, then $p|a$ or $p|b$.

(b) Suppose that a and b are non-zero integers. Then prove that $\text{lcm}(a,b) = \dfrac{|ab|}{\gcd(a,b)}$.

(c) Prove that every integer $n > 1$ can be written as a product of primes.

(d) Suppose that $a = 341$ and $b = 527$. Find $\gcd(a,b)$ and $\text{lcm}(a,b)$.