EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE 1996/97

(June/July' 2004) (Repeat)

EXTERNAL DEGREE

EXMT 101 - FOUNDATION OF MATHEMATICS

Answer four questions only

Time : Two hours

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1. (a) i. Define the following terms "tautology" and "contradiction" as applied to a proposition.

   ii. Explain what is meant by the statement that two propositions are logically equivalent.

   (b) Let p, q and r be three propositions. Prove the following:

   i. $(p \wedge q) \vee > p \equiv > p \vee q$;

   ii. $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$;

   iii. $p \rightarrow (q \rightarrow r) \equiv (p \wedge > r) \rightarrow > q$.

   (c) Test the validity of the following argument:

   If I study, then I will not fail Mathematics. If I do not play basketball, then I will study. But I failed Mathematics. Therefore, I played basketball.

2. Define the following:

- The difference, $A \setminus B$, of two sets A and B,
- Symmetric difference, $A \triangle B$, of two sets A and B.

Prove the following:

(a) $A \cup B = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$;

(b) $(A \cap B) \setminus (A \cap C) = A \cap (B \setminus C)$;

(c) $(A \triangle B) = (A \cup B) \setminus (A \cap B)$;

(d) $(A \triangle B) \cap (A \cap B) = \phi$.

3. What is meant by an equivalence relation?

(a) Let R be the relation in the natural numbers such that $xRy \Leftrightarrow (x - y)$ is divisible by 5. Prove that R is an equivalence relation.

(b) Let A be a set and let $\sim$ be an equivalence relation on A. Let $[a] = \{x \in A \mid x \sim a\}$. Prove the following:

i. $[a] \neq \Phi \quad \forall a \in A$;

ii. $a \sim b \Leftrightarrow [a] = [b] \quad \forall a, b \in A$;

iii. $b \in [a] \Leftrightarrow [a] = [b] \quad \forall a, b \in A$;

iv. Either $[a] = [b]$ or $[a] \cap [b] = \Phi \; \forall a, b \in A$.

4. (a) Define the following terms.

   i. Injective function;

   ii. Surjective function;

   iii. Bijective function.

   (b) Let $f : S \longrightarrow T$ be a function and let A, B be subsets of S.

   i. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.

   ii. Prove that $f(A \cup B) = f(A) \cup f(B)$.

   iii. Is it true that $f(A) \cap f(B) \subseteq f(A \cap B)$? Justify your answer.

   (c) Let $f : A \longrightarrow B$ and $g : B \longrightarrow A$ be two mappings such that $g \circ f = I_A$ and $f \circ g = I_B$. Prove that f is bijective and $g = f^{-1}$.

5. (a) Define the following terms:

   - Partially ordered set;
   - Totally ordered set;
   - First element of a partially ordered set;
   - Last element of a partially ordered set ;
   - Minimal element of a totally ordered set.

   (b) Let X be the set of all functions from $\mathbb{R}$ into $[0, 1]$. Define a relation $\sim$ on X by

$$f \sim g \Leftrightarrow f(x) - g(x) \geq 0 \quad \text{for any } f, g \in X \text{ and for every } x \in \mathbb{R}.$$

   Prove that $(X, \sim)$ is a partially ordered set.

(c) Show that if R defines a partial order on a set A then R^{-1} also defines a partial order on A.

(d) Show that if totally ordered set $(A, \preceq)$ has a minimal element then it will be the first element.

6. (a) Define the following:

   i. Group,

   ii. Subgroup of a group.

(b) Let G be the set of real numbers except -1. An operation $\odot$ is defined on G as

$$a \odot b = a + b + ab, \quad \forall\, a, b \in G.$$

Prove that $(G, \odot)$ is a group.

(c) Let S be a subset of a group G. Prove that S is a subgroup of G if and only if the following conditions hold.

   i. $S \neq \phi$;

   ii. $x^{-1}y \in S$ for any $x, y \in S$.

(d) Prove that if H and K are subgroups of a group G then $H \cap K$ is also a subgroup of G.