EASTERN UNIVERSITY, SRI LANKA

IRST YEAR EXAMINATION IN ARTS AND CULTURE (2010/2011)

JUNE' 2011

CC 101 - BASIC MATHEMATICS

EXTERNAL DEGREE

nswer only Five questions   Time: Three hours

1. (a) Simplify the following expressions:

   i. $\dfrac{2x^2y}{3yz} \times \dfrac{5z^2x}{7xy^2} \div \dfrac{21x^2y^3z^2}{40xy^2z}$;

   ii. $\dfrac{a^3}{b^3} \times \dfrac{xy^2}{ab} \times \dfrac{pb^2}{ax} \div \dfrac{ap}{b^2}$;

   iii. $\log\left\{\left(\dfrac{x^4y^{-3}}{x^{-1}y^2}\right)^{-3} \div \left(\dfrac{x^{-2}y^3}{xy^{-1}}\right)^5\right\}$.

   (b) Solve the following equations:

   i. $x - \dfrac{x-13}{9} = \dfrac{6x+1}{5} + \dfrac{2}{3}\left(6 - \dfrac{3x}{2}\right)$;

   ii. $\sqrt{3^x} + \sqrt{3^{2-x}} = 6\sqrt{3^{-x}}$.

2. (a) Factorize the following expressions:

   i. $2xy - 4xyz + 2xyz^2$;

   ii. $pq(l^2+m^2) + lm\,(p^2+q^2)$;

   iii. $(x+2y)\,(x-2y) + z\,(4y-z)$.

   (b) If $v = w\sqrt{a^2 - x^2}$, then find a.

   (c) If $p = q^{2a}$, $q = r^{2b}$ and $r = p^{2c}$, then prove that $abc = \dfrac{1}{8}$.

   (d) If $x^2 + y^2 = 7xy$, then prove that $\log(x+y) = \log 3 + \frac{1}{2}\log x + \frac{1}{2}\log y$.

3. (a) Solve the following simultaneous equations:

   $$\frac{5}{x} + \frac{16}{y} = 79,$$
   $$\frac{16}{x} - \frac{1}{y} = 44.$$

(b) i. Show that $2b^2 = 9ac$, if one solution is double of the other, of a quadratic equation $ax^2 + bx + c = 0$.

ii. If α and β are the solutions of the quadratic equation $x^2 + 6x - 7 = 0$, then find the quadratic equation for which $\frac{1}{\alpha+1}, \frac{\alpha\beta}{\alpha+\beta}$ are solutions.

4. (a) Write the order of the following matrices.

i. $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$;

ii. $\begin{pmatrix} 4 & 0 \\ 1 & 0 \\ 10 & 0 \end{pmatrix}$;

iii. $\begin{pmatrix} 1 & 3 & 0 & 1 \\ -3 & 9 & 2 & 6 \\ 3 & 5 & 6 & 8 \end{pmatrix}$.

(b) Find the $(i,j)^{\text{th}}$ entry of a 3×3 matrix $A = (a_{i,j})$, if

i. $a_{ij} = i + j$;

ii. $a_{ij} = i \times j$.

(c) Find x, y, z and w, if $\begin{pmatrix} 2x & 3x - y \\ 2x + z & 3y - w \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 7 \end{pmatrix}$.

(d) Solve the following matrix equations:

$$2X + Y + \begin{pmatrix} -2 & 1 & 3 \\ 5 & -7 & 3 \\ 4 & 5 & 4 \end{pmatrix} = 0,$$

$$X - Y = \begin{pmatrix} 4 & 7 & 0 \\ -1 & 2 & -6 \\ -2 & 8 & -5 \end{pmatrix},$$ where X and Y are 3×3 matrices.

(e) Verify the matrix equation $(A+B)^2 = A^2 + AB + BA + B^2$, for $A = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}$.

5. (a) Find the following limits:
   i. $\lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$;
   ii. $\lim_{x\to 1} \dfrac{x^2+2x+5}{x^2+1}$;
   iii. $\lim_{x\to 4} \dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

   (b) Differentiate the following functions with respect to x :
   i. $\dfrac{x}{\sqrt{x+2}}$;
   ii. x^2e^x.

   (c) i. Evaluate the following:
   A. $\lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$;
   B. $\lim_{x\to 1} \dfrac{x^2+2x+5}{x^2+1}$;
   C. $\lim_{x\to 4} \dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

   ii. Differentiate the following with respect to x :
   A. $\dfrac{x}{\sqrt{x+2}}$;
   B. x^2e^x.

   iii. A. Find $\dfrac{dy}{dx}$ in terms of t, if $x = \dfrac{t}{1-t}$ and $y = \dfrac{t^2}{1-t}$.
   B. Analyze the turning points of the function $y = 4x^3 + 9x^2 - 12x + 3$.

6. (a) Integrate the following with respect to x.
   i. $\displaystyle\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$;
   ii. $\displaystyle\int \frac{2x+1}{3x^2+3x+1} dx$;
   iii. $\displaystyle\int \frac{e^{3x}}{e^{3x}+3} dx$.

   (b) Evaluate the following:
   i. $\displaystyle\int_2^3 \frac{x}{x+x^2} dx$;
   ii. $\displaystyle\int_1^2 (x^2+3x-5)\, dx$.

7. (a) i. Show that the points $(3,-4)$, $(9,4)$ and $(12,8)$ lie on a straight line and find its equation.

ii. Find the equation of the straight line passes through $(1, 2)$ and perpendicular to $2y = 3x + 2$.

(b) There are 40 students in a class. Most of them offered Maths, Science and English as subjects. Two of them offered non of the given subjects. 20 students offered Science, 26 offered Maths. 12 students offered all three subjects. Among those offered English as a subject 17 offered Maths and among those offered Science as a subject 13 offered English. 6 of the students in the class offered Maths only. Find the number of students who offered

i. only Science.

ii. only English.

iii. only Science and Maths.

iv. English and Maths but not Science.

8. (a) A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of drawing:

i. a king or a queen;

ii. a king or a spade;

iii. a king or a black card.

(b) If A and B are any two events and $\overline{B}$ is the complimentary even of B.
If $P(A) = 0.35$, $P(B) = 0.73$ and $P(A \cap B) = 0.14$. Find the following:

i. $P(A \cup B)$;

ii. $P(\overline{A} \cap B)$;

iii. $P(A \cap \overline{B})$;

iv. $P(\overline{A} \cup \overline{B})$;

v. $P(\overline{A \cup B})$.

Hint: You may assume the following probability identities.

i. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

ii. $P(A \cap \overline{B}) = P(A) - P(A \cap B)$.

9. Following data obtained from a rubber estate regarding the perimeter of the rubber trees.

Class interval (perimeter(cm))	Frequency(no of trees)
35-40	18
40-45	58
45-50	101
50-55	65
55-60	41
60-65	32
65-70	23
70-75	18

(a) Find the modal and median classes;

(b) calculate the mean perimeter of the rubber trees by guessed mean as the mid point of median class.

(c) Find the percentage of the trees which have greater perimeter than the mid point of the clsass $50 - 55$.