EASTERN UNIVERSITY, SRI LANKA
EXTERNAL DEGREE EXAMINATION IN SCIENCE
SECOND YEAR FIRST SEMESTER - 2003/2004, 2004/2005(July/August, 2008)
DEPARTMENT OF MATHEMATICS
EXTMT 207 - NUMERICAL ANALYSIS
(PROPER & REPEAT)

Answer all Questions **Calculators are provided** **Time: Two hours**

Q1. (a) By writing any real number p in a normalized decimal form, explain the terms "chopping" and "rounding".

(b) In a floating point number system, prove that

$$|\text{relative round-off error}| \leq \begin{cases} \beta^{1-t}, & \text{for chopping;} \\ \frac{1}{2}\,\beta^{1-t}, & \text{for rounding,} \end{cases}$$

where β and t denote the base number and a maximum number of decimal digits, respectively.

(c) If three approximated values of the number $\frac{1}{3}$ are 0.30, 0.33 and 0.34, which of these is the best approximation?

Q2. (a) Let $x = g(x)$ be an arrangement of the equation $f(x) = 0$, which has root α in the interval I. If $g'(x)$ exists and continuous in I satisfying

$$|g'(x)| \leq h < 1, \ \forall x \in I,$$

prove that, for any given x_0, the sequence $\{x_r\}$, $r = 0, 1, 2, \ldots$, defined by

$$x_{r+1} = g(x_r)$$

converges to α and such α is unique.

Hence, find the condition for the convergence of Newton-Raphson method.

(b) Find a root of the equation $x^3 - x - 1 = 0$ correct to four decimal places usi

Newton-Raphson and Secant methods. Compare the result you have obtaine

Q3. (a) Write down the Lagrange's interpolation formula and show that such interp lation formula p of $f : [a, b] \longrightarrow \mathbb{R}$ satisfying $p(x_i) = f(x_i),\ i = 0, 1, \ldots,$ where x_i 's are distinct in $[a, b]$, always exists and is unique.

(b) Prove that the error in Lagrange's interpolation has the form

$$\frac{(x - x_0)(x - x_1)(x - x_2)\ldots(x - x_n)}{(n+1)!} f^{n+1}(\xi), \quad \xi \in (a, b).$$

(c) Find the Lagrange's interpolating polynomial of degree 2 approximating t function $y = \ln x$ using the following tabular values. Hence, find the value ln(2.7) and determine the error using part (b).

x	2.00000	2.50000	3.00000
$\ln x$	0.69315	0.91629	1.09861

Q4. (a) Write down the formula for the integral

$$\int_{x_0}^{x_1} f(x)\, dx$$

and its error term, which represent the trapezoidal rule in the interval $[x_0, x_1$ Hence, derive the composite form of the trapezoidal rule and its error ter over the interval $[a, b]$.

(b) Use composite trapezoidal to evaluate an approximate value of

$$\int_0^1 \frac{1}{1+x}\, dx$$

correct to three decimal places using the following table:

x	0.0000	0.2500	0.5000	0.7500	1.0000
y	1.0000	0.8000	0.6667	0.5714	0.5000

Estimate truncation and round-off errors.

(c) Consider the following system of linear equations:

$$
\begin{aligned}
10x_1 - 2x_2 - x_3 - x_4 &= 3 \\
-2x_1 + 10x_2 - x_3 - x_4 &= 15 \\
-x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\
-x_1 - x_2 - 2x_3 + 10x_4 &= -9.
\end{aligned}
$$

Use Gauss-Seidel method to carry out 3 iterations for x_1, x_2, x_3 and x_4 correct to 4 decimal places.